Let Q be the point $(-1, 4, -4)$, R be the point $(3, -1, 3)$, and P be the point such that $\overrightarrow{PQ}$ is the vector $3\vec{j} - 3\vec{k}$.

ALL ITEMS 4 POINTS UNLESS OTHERWISE NOTED

SCORE: ____ / 100 PTS

[a] If $\vec{v}$ is a vector of magnitude 8, and the angle between $\overrightarrow{PQ}$ and $\vec{v}$ is $\frac{5\pi}{6}$ radians, find $\overrightarrow{PQ} \cdot \vec{v}$.

$$\|\overrightarrow{PQ}\| \|\vec{v}\| \cos\theta = (3\sqrt{2})(8)\cos\frac{5\pi}{6} \quad 6$$
$$= (3\sqrt{2})(8)\left(-\frac{\sqrt{3}}{2}\right)$$
$$= -12\sqrt{6}$$

$$\|\overrightarrow{PQ}\| = \sqrt{0^2 + 3^2 + (-3)^2}$$
$$= \sqrt{18}$$
$$= 3\sqrt{2}$$

[b] In which octant is P ?

$$\langle -1-x, 4-y, -4-z \rangle = \langle 0, 3, -3 \rangle$$
$$-1-x=0 \qquad x=-1$$
$$4-y=3 \longrightarrow y=1$$
$$-4-z=-3 \qquad z=-1$$

$(-1, 1, -1)$ IS IN $O_{2+4} = O_6$

[c] Find a vector of magnitude 8 in the opposite direction as $\overrightarrow{PR}$.

$$\overrightarrow{PR} = \langle 3-{}^-1, -1-1, 3-{}^-1 \rangle = \langle 4, -2, 4 \rangle$$

$$\frac{-8}{\|\langle 4,-2,4\rangle\|}\langle 4,-2,4\rangle = \frac{8}{\sqrt{4^2+(-2)^2+4^2}}\langle 4,-2,4\rangle \quad 7$$
$$= \frac{8}{\sqrt{36}}\langle 4,-2,4\rangle$$
$$= \frac{\cancel{-8}^{-4}}{\cancel{6}_{3}}\langle 4,-2,4\rangle$$
$$= \left\langle -\frac{16}{3}, \frac{8}{3}, -\frac{16}{3} \right\rangle$$

[d] If $2\vec{i} - \vec{j} - c\vec{k}$ is perpendicular to $\overrightarrow{PR}$, find the value of c.

$$\langle 2, -1, -c \rangle \cdot \langle 4, -2, 4 \rangle = 0 \quad 6$$
$$8 + 2 - 4c = 0$$
$$c = \frac{5}{2}$$

[e] Find the volume of the parallelepiped with $\overrightarrow{PQ}$, $\overrightarrow{PR}$ and $<2, 1, -1>$ as adjacent edges.

$$\left|\begin{matrix} 0 & 3 & -3 \\ 4 & -2 & 4 \\ 2 & 1 & -1 \end{matrix}\right| \begin{matrix} 0 & 3 \\ 4 & -2 \\ 2 & 1 \end{matrix} = 0+24-12-(12+0-12) = 12$$

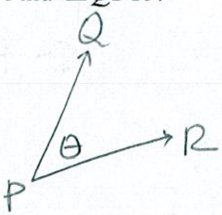
6

5

[f] If you start at point P, move 2 units to the left, 4 units down, and 6 units forward, find the co-ordinates of your ending point.

$$(-1+6, 1-2, -1-4) = (5, -1, -5)$$

[g] Find $\angle QPR$.



$$\cos^{-1} \frac{\overrightarrow{PQ} \cdot \overrightarrow{PR}}{\|\overrightarrow{PQ}\| \|\overrightarrow{PR}\|} = \cos^{-1} \frac{\langle 0, 3, -3 \rangle \cdot \langle 4, -2, 4 \rangle}{(3\sqrt{2})(6)}$$

6

$$= \cos^{-1} \frac{0-6-12}{18\sqrt{2}}$$

$$= \cos^{-1} \frac{-\cancel{18}}{\cancel{18}\sqrt{2}}$$

$$= \cos^{-1} -\frac{\sqrt{2}}{2}$$

$$= \frac{3\pi}{4} \text{ OR } 135^\circ$$

Fill in the blanks. **SCORE: ____ / 10 PTS**

3

2

[a] The equation of the z-axis is $x = y = 0$ and the equation of the yz-plane is $x = 0$.

3

[b] If $\vec{w} \cdot \vec{w} = 12$, then $\vec{w} \times \vec{w} =$ $\vec{0}$ and $\|\vec{w}\| =$ $2\sqrt{3}$.

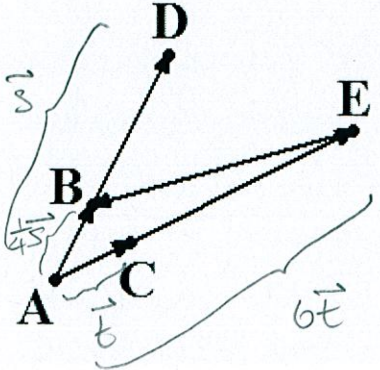
$\|\vec{w}\|^2 = 12$

2

In the diagram below, ABD and ACE are both line segments. **SCORE: ____ / 10 PTS**

AE is six times the length of AC, and BD is three times the length of AB.

If $\vec{s} = \overrightarrow{AD}$ and $\vec{t} = \overrightarrow{AC}$, find an expression for $\overrightarrow{EB}$ in terms of $\vec{s}$ and $\vec{t}$.



$$6\vec{t} + \overrightarrow{EB} = \frac{1}{4}\vec{s}$$

$$\overrightarrow{EB} = \underbrace{\frac{1}{4}\vec{s}}_{4} - \underbrace{6\vec{t}}_{2}$$

4

4

Let $\wp_1$ be the plane $2x - 3z = 8$, $\vec{n}_1 = \langle 2, 0, -3 \rangle$ **SCORE: ____ / 30 PTS**

and let $\wp_2$ be the plane $3x - y - 5z = 6$. $\vec{n}_2 = \langle 3, -1, -5 \rangle$

Let ℓ_1 be the line which passes through $(0, 6, -4)$ and is parallel to both $\wp_1$ and $\wp_2$.

Let ℓ_2 be the line which passes through $(-4, -8, 6)$ and is parallel to ℓ_1.

Let $\wp_3$ be the plane which passes through $(2, -7, 9)$ and is perpendicular to the parallel lines ℓ_1 and ℓ_2.

[a] Find parametric equations for line ℓ_1.

$\vec{d}_1 \perp \vec{n}_1$ AND $\vec{n}_2$

$$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 0 & -3 \\ 3 & -1 & -5 \end{vmatrix} \begin{matrix} \vec{i} & \vec{j} \\ 2 & 0 \\ 3 & -1 \end{matrix}$$ 3

$$= -9\vec{j} - 2\vec{k} - (3\vec{i} - 10\vec{j}) = \langle -3, 1, -2 \rangle \rightarrow \text{USE } \vec{d}_1 = \langle 3, -1, 2 \rangle$$ 3 1

3

$$\boxed{\begin{aligned} x &= 3t \\ y &= 6 - t \\ z &= -4 + 2t \end{aligned}}$$ 3

SANITY CHECK: 2

$\langle -3, 1, -2 \rangle \cdot \langle 2, 0, -3 \rangle = -6 + 0 + 6 = 0$ ✓

$\langle -3, 1, -2 \rangle \cdot \langle 3, -1, -5 \rangle = -9 - 1 + 10 = 0$ ✓

[b] Find symmetric equations for line ℓ_2. $\vec{d}_2 \parallel \vec{d}_1 \rightarrow$ USE $\vec{d}_2 = \vec{d}_1$

3

$$\frac{x+4}{3} = \frac{y+8}{-1} = \frac{z-6}{2}$$

3

$$\frac{x+4}{3} = -y-8 = \frac{z-6}{2}$$

1

[c] Find the standard (point-normal) equation for plane $\wp_3$. $\vec{n}_3 \parallel \vec{d}_1, \vec{d}_2 \rightarrow$ USE $\vec{n}_3 = \vec{d}_1 = \vec{d}_2$

3

$$3(x-2) - (y+7) + 2(z-9) = 0$$ 2

3